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On the Topology of Planar Algebraic Curves

We revisit the problem of computing the topology and geometry of a real algebraic plane curve. The topology is of prime interest but geometric information, such as the position of singular and critical points, is also relevant. A challenge is to compute efficiently this information for the given coordinate system even if the curve is not in generic position.

Previous methods based on the cylindrical algebraic decomposition (CAD) use sub-resultant sequences and computations with polynomials with algebraic coefficients. A novelty of our approach is to replace these tools by Gröbner basis computations and isolation with rational univariate representations. This has the advantage of avoiding computations with polynomials with algebraic coefficients, even in non-generic positions. Our algorithm isolates critical points in boxes and computes a decomposition of the plane (which is not a CAD) by rectangular boxes. This decomposition also induces a new approach for computing an arrangement of polylines isotopic to the input curve. We also present an analysis of the complexity of our algorithm. An implementation of our algorithm demonstrates its efficiency, in particular on high-degree non-generic curves.